

# Markets and Their Algorithmic Issues



# Markets and Prices

- Consider capitalistic economy – with set of goods and buyers
- How should prices of the goods be determined to ensure e.g. stability, fairness, and efficiency?
- Mathematical economics
  - General equilibrium theory
  - Arrow & Debreu (1954) – existence of equilibrium in general model of economy
  - Non-algorithmic theory



# Markets and Prices

- Massive computational power available for running new markets
- Need for algorithmic theory of market equilibria
- Algorithms can help understand repercussions to existing prices, production, and consumption caused by technological advances, introduction of new goods, or changes to the tax structure



# Equilibrium Price

- Stability of prices – demand must equal supply
- If there is only one good, equilibrium price is easy to determine: where demand and supply curves intersect
  - Arrow & Debreu (1954) – existence of equilibrium prices in general model of economy
- It turns out that equilibria for several market models can be captured as optimal solutions to certain nonlinear convex programs
  - Combinatorial algorithms for solving these convex programs



# A Simple Market



# Simple Market

- A – set of goods; B – set of buyers
- Buyer  $i$  has money  $m_i$ . Each good  $j$  has amount  $a_j$
- Buyer  $i$  has access to subset  $S_i$  of goods
- Once the prices  $p_1, \dots, p_n$  are fixed, a buyer is only interested in the cheapest goods  $S_i'$
- Any allocation from  $S_i'$  that exhausts her money is an *optimal basket of goods* at these prices
- Prices are *market clearing* or *equilibrium* if each buyer can be assigned an optimal basket such that there is no surplus or deficiency of any good

