Homework 1

CIS4930/5930

Due in-class, Friday Sep. 13

Instructions: Graduate students must complete all problems. Undergraduates may receive extra credit for working problems marked with (*). Show all work for full credit. Partial credit for incomplete / incorrect arguments will be given at the discretion of the instructor / TA.

1. (5 points) Greatest lower bound. A *lower bound* of a set $A \subseteq \mathbb{R}$ is a number γ such that for all $a \in A$, $\gamma \leq a$. The *infimum*, or *greatest lower bound* of A is a lower bound γ, such that for any lower bound γ', it holds that $\gamma' \leq \gamma$. The notion of upper bound and least upper bound or *supremum* is defined analogously.

   - (1 point) Compute $\inf\{1/n : n \in \mathbb{N} \setminus \{0\}\}$.
   - (1 point) Compute $\sup\{1/n : n \in \mathbb{N} \setminus \{0\}\}$.
   - (3 points) Show that $\inf A = -\sup\left(\{-a : a \in A\}\right)$.

2. (5 points) For binary classification with loss function $L(y', y) = 1_{y' \neq y}$, recall that
$$noise(x) = \min\{Pr[1|x], Pr[0|x]\},$$
and that the Bayes error R^* is defined by
$$R^* = \inf_h R(h).$$
Show that $E[noise(x)] = R^*$.

3. (2 points) Show that if $Pr[X \geq \epsilon] \leq f(\epsilon)$ for some invertible function $f : \mathbb{R}_+ \to \mathbb{R}_+$, then, for any $\delta > 0$, with probability at least $1 - \delta$, $X \leq f^{-1}(\delta)$.

4. (2 points) Prove Chebyshev's Inequality by following the outline in the slides for lecture 2 (σ^2 denotes the variance of the random variable X).

5. (5 points) MRT, exercise 2.2.

6. (*) (5 points) MRT, exercise 2.8