Exam 1

CIS4930/5930

Instructions: Undergraduates work any three problems; graduate students work any four problems. Clearly indicate which problems you want graded. Show all work for full credit. Partial credit for incomplete / incorrect arguments will be given at the discretion of the instructor / TA.

1. Concentric circles. Consider the set of concepts of the form $c = \{(x, y) : x^2 + y^2 \le r^2\} \subseteq \mathbb{R}^2$ for some real number r. Show that this class can be (ϵ, δ)-PAC-learned from training data of size $m \ge (1/\epsilon)\log(1/\delta)$.

2. Show that the VC-dimension of a finite hypothesis set H is at most $\log_2 |H|$.

3. Soft margin hyperplanes. The function of the slack variables used in the optimization problem for soft margin hyperplanes has the form: $\boldsymbol{\xi} \mapsto \sum_{i=1}^m \xi_i$. Suppose this is replaced by $\boldsymbol{\xi} \mapsto \sum_{i=1}^m \xi_i^p$, with $p > 1$. Give the dual formulation of the problem in this general case.

4. Show that $K(x, y) = \cos(x - y)$ is a PDS kernel.

5. Let $\mathcal{H}$ be any hypothesis class and let $g : X \to \mathbb{R}$. Define $H + g^2 = \{x \mapsto h(x) - g^2(x) : h \in H\}$. Show that $\mathcal{R}_m(\mathcal{H}) = \mathcal{R}_m(\mathcal{H} + g^2)$.