#### Introduction to Game Theory



#### Game Theory

- Mathematical study of interaction among independent, self-interested agents
- Applied to economics, political science, biology, psychology, linguistics, and to computer science
- Noncooperative games
- Normal-form games



#### Self-Interested Agents

- Each agent has her own description of which states or outcomes she likes
- How to model such interests or preferences?
- Utility theory
- Utility function



### Example: friends and enemies

- Alice: going to club (*c*), going to a movie (*m*), staying home (*h*)
- By herself, Alice has the utility values of: u(c) = 100, u(m) = 50, u(h) = 50.
- But Alice's utility changes based on other agents: Bob and Carol
- Bob at movies: disutility of 40. Bob at club: disutility of 90.
- Carol: multiply utility by 1.5



### Example: friends and enemies

- Bob at movies: disutility of 40. Bob at club: disutility of 90.
- Carol: multiply utility by 1.5
- Bob at club 60% of time, 40% at movies
- Carol at club 25% of time, movies 75% of time



### Example: friends and enemies

s	B = c	B = m		B = c	B = m
C = c	15	150	C = c	50	10
C = m	10	100	C = m	75	15
A = c				<i>A</i> =	= m

Figure 3.1: Alice's utility for the actions c and m.

- Expected utility for A=c: 51.75
- Expected utility for A=m: 46.75
- So Alice prefers to go the club even thous Bob is often there and Carol rarely is.

# Games in normal form



#### Games

- Under reasonable assumptions about preferences, agents will have utility functions they want to maximize
- Simple if one agent (like in Alice example previously)
- But what if, instead of acting probabilistically, Bob hates Alice and wants to avoid her too?
- And Carol is indifferent to seeing Alice and has a crush on Bob?



- Imagine you are colleague are only two users on a department network
- Traffic governed by the TCP protocol, which has backoff mechanism when encountering congestion
- Two strategies: Use a correct implementation of TCP (*C*) or defective one (*D*)



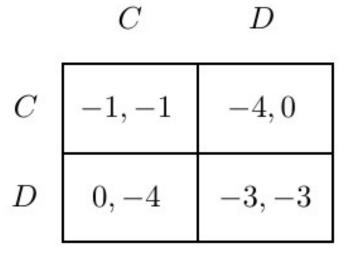


Figure 3.3: The TCP user's (aka the Prisoner's) Dilemma.

• Given these outcomes, which should you adopt, *C* or *D*?



- Does your choice depend on what you think your colleague will do?
- From perspective of network operator, what kind of behavior could you expect?
- Under what changes to the delay would the users' decisions still be the same?
- Do answers depend on rationality and / or perception of rationality?



- Game theory gives answers to many of these questions.
- Any rational user would adopt *D* if playing once or even if playing multiple times
- However, if number of times playing is uncertain or infinite, we may see the users adopt *C*



#### Definition: Normal-Form Game

- Representation of every player's utility for every state of the world
- States of the world only depend on players' combined actions
- More general settings (world depends on randomness in environment as well as players' actions) can be reduced to normal-form games
- Also normal-form reductions for other game representations



#### Definition: Normal-Form Game

**Definition 3.2.1 (Normal-form game)** A (*finite, n-person*) normal-form game is a tuple (N, A, u), where:

- N is a finite set of n players, indexed by i;
- $A = A_1 \times \cdots \times A_n$ , where  $A_i$  is a finite set of actions available to player *i*. Each vector  $a = (a_1, \ldots, a_n) \in A$  is called an action profile;
- $u = (u_1, \ldots, u_n)$  where  $u_i : A \mapsto \mathbb{R}$  is a real-valued utility (or payoff) function for player i.



#### Example - Prisoner's Dilemma

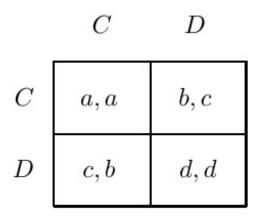


Figure 3.4: Any c > a > d > b define an instance of Prisoner's Dilemma.



#### Example - Commonpayoff games

**Definition 3.2.2 (Common-payoff game)** A common-payoff game is a game in which for all action profiles  $a \in A_1 \times \cdots \times A_n$  and any pair of agents i, j, it is the case that  $u_i(a) = u_j(a)$ .

Right

	A		
Left	1, 1	0, 0	
Right	0, 0	1,1	

Left

Figure 3.5: Coordination game.



#### Example - Zero-Sum Games

**Definition 3.2.3 (Constant-sum game)** A two-player normal-form game is constantsum if there exists a constant c such that for each strategy profile  $a \in A_1 \times A_2$  it is the case that  $u_1(a) + u_2(a) = c$ .

	Heads	Tails
Heads	1, -1	-1, 1
Tails	-1, 1	1, -1

Figure 3.6: Matching Pennies game.



### Example - Coordination and Competition

#### Husband



Wife	LW	2,1	0, 0
whe	WL	0, 0	1,2

Figure 3.8: Battle of the Sexes game.



### Strategies in normalform games



#### Strategies

- Recall definition of normal-form game. Each player has a utility function that depends on the action profile selected
- Each player can select an action from her available actions and play it *pure strategy*
- Each player can also select a probability distribution over some set of their actions and play randomly according to that distribution *mixed strategy*



#### Strategies

**Definition 3.2.4 (Mixed strategy)** Let (N, A, u) be a normal-form game, and for any set X let  $\Pi(X)$  be the set of all probability distributions over X. Then the set of mixed strategies for player i is  $S_i = \Pi(A_i)$ .

**Definition 3.2.5 (Mixed-strategy profile)** The set of mixed-strategy profiles is simply the Cartesian product of the individual mixed-strategy sets,  $S_1 \times \cdots \times S_n$ .

**Definition 3.2.6 (Support)** The support of a mixed strategy  $s_i$  for a player *i* is the set of pure strategies  $\{a_i | s_i(a_i) > 0\}$ .

**Definition 3.2.7 (Expected utility of a mixed strategy)** Given a normal-form game (N, A, u), the expected utility  $u_i$  for player i of the mixed-strategy profile  $s = (s_1, \ldots, s_n)$  is defined as

$$u_i(s) = \sum_{a \in A} u_i(a) \prod_{j=1}^n s_j(a_j).$$

